

Problem Sheet 3 – submit by August 24th

Instructor: David Freeborn

Question 1: Consider the following chance setup. You have a hat with 10 cards in it. Five of these cards are marked with the letter A. Four are marked with the letter B. One is marked with the letter C. You also have three jars, labeled A, B, and C, which have differently colored balls as given in the following table.

Colors	Jar A	Jar B	Jar C
Green	2	3	0
Yellow	2	0	5
Red	0	6	3
Black	4	0	0

A trial of this chance setup proceeds as follows. First, you draw a card at random. If you have drawn an A, then you draw a ball from jar A. If you have drawn a B, then you draw a ball from urn B. If you have drawn a C, then you draw a ball from urn C.

(a) List the possible outcomes (combinations of the result of the card draw and color of ball drawn). [*Hint: a tree diagram helps think this through.*]

(b) What is the probability of drawing a black ball?

(c) What is the probability of drawing a ball that is not black, given that you draw a card marked C?

(d) What is the probability of drawing a ball that is *not yellow*?

(e) Suppose I drew a yellow ball. What is the probability that I drew it from urn A?

Question 2: The small town of Sunnydale, California (population 300) is being overrun by the Tarellian Virus. Of the population, 244 people have been vaccinated, 56 have not been vaccinated. 99 people have been infected in total. Of those infected, 45 people have been vaccinated, and 44 people have not been vaccinated.

(a) Assuming there are no other factors at play, what is the conditional probability of becoming infected if you are vaccinated, and if you are not vaccinated?

(b) Are those reasonable assumptions?

(c) Famous YouTuber, Keith Oddlebloggle, claims that the vaccine must be totally ineffective, because more vaccinated people have been infected than unvaccinated people. Is he right?

Question 3: Professor Grumble is suspicious that Alice took *Wakium*, an illegal, performance-enhancing drug during her classes. He notes that Alice is normally only awake in around $\frac{1}{4}$ of classes. But he thinks Alice is the type of person who might take a drug like *Wakium* with about 20% probability. And he thinks if Alice took *Wakium*, she would be awake about 60% of classes. Given that he saw that Alice was awake in today's class, with what probability does Professor Grumble conclude that Alice took *Wakium* during her exam?

Question 4: In a particular pain clinic, 10% of patients are prescribed narcotic pain killers. Overall, 5% of the clinic's patients are addicted to narcotics (including pain killers). Out of all the people prescribed pain pills, 8% are addicts. If a patient is an addict, what is the probability that they will be prescribed pain killers?

Question 5: Professor Freeborn has lost Brianne's homework again. Tsk. He doesn't want to admit losing the homework, so he decides to make up a grade for Brianne. So Professor Freeborn needs to decide whether to give Brianne an *A* grade or not. He makes the following observations. 30% of his students get an *A*. But of those who get an *A*, 90% hand their homework in on time every week. Of those who don't get an *A*, only 40% hand their homework in on time every week. Professor Freeborn notes that Brianne always hands her homework in on time (it's just a shame he keeps losing it...). So what's the probability that Brianne should get an *A*?